XLVI. *Problems by* Edward Waring, *M. A. and* Lucaſian *Profeſſor of Mathematics in the Univerſity of* Cambridge, *F. R. S.*

PRO.

Read April 21, 1763.

1. INvenire, quot radices impoſſibiles habet data biquadratica æquatio $x^4+qx^2-rx+s=0$.

1^mo^ Sit $256s^3-128q^2s^2+\overline{144r^2q+16q^4}\times s-27r^4-4r^2q^3$ negativa quantitas, & duas & non plures impoſſibiles radices habet data æquatio.

2^do^ Sit affirmativa quantitas, & vel $-q$ vel q^2-4s negativa quantitas, & datæ æquationis quatuor radices erunt impoſſibiles.

3^tio^. Sit nihilo æqualis, & vel $-q$ vel q^2-4s negativa quantitas, & datæ æquationis duæ inæquales radicis erunt impoſſibiles.

2. Invenire, quot radices impoſſibiles habet data æquatio $x^5+qx^3-rx^2+sx-t=0$.

1^mo^ Si ſigna terminorum æquationis

$$w^{10}+10qw^9+\overline{39q^2+10s}\times w^8+\overline{80q^3+50qs+25r^2}\times w^7+\overline{95q^4+124q^2s-95s^2+92qr^2+200rt}\times w^6+\overline{66q^5-360qs^2+196q^3s+118q^2r^2+260r^2s+625t^2+400qrt}\times w^5+\overline{25q^6+40s^3-53r^4+52q^3r^2-522q^2s^2+194q^4s+708qr^2s+240q^2rt+1750qt^2-950srt}\times w^4+\overline{4q^7+106q^5s-80qs^3-308q^3s^2-102qr^4-7q^4r^2+570r^2s^2+612q^2r^2s+700r^2t-3750t^2s+2500t^2q^2+80rtq^3-2150qrst}\times w^3+400s^4-360q^2s^3-15q^4s^2+24q^6s-8q^5r^2-45$$

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$-45 q^2 r^4 - 270 r^4 s + 140 r^2 s q^3 + 960 r^2 s^2 q + 1875$
$t^2 r^2 + 1000 t r s^2 - 5000 t^2 q s + 1750 t^2 q^3 + 40 t r q^4$
$+ 600 t r^3 q - 1650 t r s q^2 \times w^2 + 36 q^5 s^2 - 224 q^3 s^3$
$+ 320 q s^4 + 4 q^3 r^4 + 27 r^6 - 40 r^2 s^2 + 434 r^2 q^2 s^2 -$
$24 r^2 s q^4 - 198 r^4 q s + 5000 t^2 s^2 - 450 t r^3 s - 6250$
$t^3 r + 675 t^2 q^4 - 3750 t^2 q^2 s + 3000 t^2 r^2 q + 60 t r^3 q^2$
$+ 200 t r s^2 q - 330 t r q^3 s \times w + 3125 t^4 - 3750 q r t^3$
$+ 2000 s^2 q + 2250 r^2 s - 900 s q^3 + 825 r^2 q^2 + 108 q^5$
$\times t^2 - 1600 s^3 r - 560 r q^2 s^2 - 16 r^3 q^3 + 630 r^3 q s +$
$72 r s q^4 - 108 r^5 \times t + 256 s^5 - 128 q^2 s^4 + 144 r^2 q s^3$
$+ 16 q^4 s^3 - 27 r^4 s^2 - 4 r^2 q^3 s^2 = 0$. continuo mutentur de + in —; & — in +; nullas impoſſibiles radices habet data æquatio.

2do. Si ſigna terminorum æquationis haud continuo mutentur de + in — & — in +; duæ vel quatuor datæ æquationis radices erunt impoſſibiles, prout ultimus ejus terminus ſit negativa vel affirmativa quantitas.

3^{tio}. Si ultimus ejus terminus nihilo ſit æqualis, & ſigna terminorum æquationis haud continuo mutentur de + in — & — in +; tum vel quatuor vel duæ radices datæ æquationis erunt impoſſibiles, prout duo & non plures ultimi datæ æquationis termini nihilo ſint æquales, necne.

P R O.

Sint x, y, v, abſciſſa, ordinata & area datæ curvæ, & ſit $y^n + \overline{a + bx} \times y^{n-1} + \overline{c + dx + ex^2} \times y^{n-2} + \overline{f + gx + hx^2 + kx^3} \times y^{n-3} +$ &c. $= 0$: invenire, utrum area (v) quadrari poteſt, necne.

Supponamus æquationem ad aream eſſe $v^n + \overline{A + Bx + Cx^2}\, v^{n-1} + \overline{D + Ex + Fx^2 + Gx^3 + Hx^4} \times$

$v^{n-2} + \overline{I + Kx + Lx^2 + Mx^3 + Nx^4 + Ox^5 + Px^6} \times v^{n-3} + \&c. = 0.$ & conſequenter erit

$$\left.\begin{array}{l} n y v^{n-1} + \overline{n-1}\;\overline{A + Bx + Cx^2}\, y v^{n-2} + \overline{n-2} \times \overline{D \times Ex + Fx^2 + Gx^3 + Hx^4} \times y v^{n-3} + \&c. \\ \overline{B + 2Cx}\; v^{n-1} + \overline{E + 2Fx + 3Gx^2 + 4Hx^3} \times v^{n-2} + \&c. \end{array}\right\} = 0.$$

Ex quibus æquationibus, ſi methodis notis exterminetur (v), habebimus æquationem, quæ exprimit relationem inter (x) & (y). Hujus autem æquationis coefficientes æquari debent coefficientibus datæ æquationis $y^n + \overline{a + bx}\, y^{n-1} + \overline{c + dx + ex^2} + y^{n-2} + \&c. = 0$; & ſi quantitates A, B, C, &c. exinde determinari poſſunt, curva quadratur, eſt enim $v^n + \overline{A + Bx + Cx^2} \times v^{n-1} + \overline{D + Ex + Fx^2 + Gx^3 + Hx^4} \times v^{n-2} + \&c. = 0$; aliter autem quadrari non poteſt.

Ex. Sit data æquatio $y^2 + x^2 - 1 = 0$, & ſupponamus æquationem ad aream $v^2 + D + Ex + Fx^2 + Gx^3 + Hx^4 = 0$; & erit $2vy + E + 2Fx + 3Gx^2 + 4Hx^3 = 0$, ita reducantur hæ duæ æquationes in unam, ut exterminatur (v), & reſultat æquatio $y^2 + \frac{16H^2x^6 + 24HGx^5 + \overline{16HF + 9G^2}\, x^4 + \overline{8EH + 12FG}\, x^3 + \overline{6GE + 4F^2}\, x^2 + 4FEx + E^2}{4 \times \overline{Hx^4 + Gx^3 + Fx^2 + Ex + D}} = 0$; debet autem fractio $\frac{16H^2x^6 + 24HGx^5 + \overline{16HF + 9G^2}\, x^4 + \overline{8EH + 12FG}\, x^3 + \overline{6GE + 4F^2}\, x^2 + 4FEx + E^2}{4 \times \overline{Hx^4 + Gx^3 + Fx^2 + Ex + D}}$ eſſe $x^2 - 1$; & conſequenter

$$\begin{array}{rcl} 4H & = & 16H^2 \\ 4G & = & 24HG \\ 4F-4H & = & 16HF+9G^2 \\ 4E-4G & = & 8HE+12FG \\ 4D-4F & = & 6GE+4F^2 \\ -4E & = & 4FE \\ -4D & = & E^2 \end{array}$$

ſed e methodo communes diviſores inveniendi conſtat has æquationes inter ſe contradictorias eſſe, & conſequenter curvam haud generaliter eſſe quadrabilem.

THEO.

Sint x, y, v, abſciſſa & ordinatæ curvarum ABCDEFGHI &c. & $A\beta\gamma\delta\varepsilon$ &c. & ſit $y = px^n$, & $v = \frac{n}{2.3} pa^{n-1}x - \frac{n\times\overline{n-1}\times\overline{n-2}}{30\times2\times3} pa^{n-3}x^3 + \frac{n\times\overline{n-1}\times\overline{n-2}\times\overline{n-3}\times\overline{n-4}}{42\times2\times3\times4\times5} pa^{n-5}x^5 - \frac{n\times\overline{n-1}\times\overline{n-2}\times\overline{n-3}\times\overline{n-4}\times\overline{n-5}\times\overline{n-6}}{30\times2\times3\times4\times5\times6\times7} pa^{n-7}x^7 + \frac{5n\times\overline{n-1}\times\overline{n-2}\times\overline{n-3}\times\overline{n-4}\times\overline{n-5}\times\overline{n-6}\times\overline{n-7}\times\overline{n-8}}{66\times2\times3\times4\times5\times6\times7\times8\times9} pa^{n-9}x^9 - \frac{691\times n\times\overline{n-1}\times\overline{n-2}\times\overline{n-3}\times\overline{n-4}\times\overline{n-5}\times\overline{n-6}\times\overline{n-7}\times\overline{n-8}\times\overline{n-9}\times\overline{n-10}}{2730\times2\times3\times4\times5\times6\times7\times8\times9\times10\times11} pa^{n-11}x^{11} +$ &c. cujus ultimus terminus debet eſſe x^{n-1} vel x^{n-2}, prout (n) eſt par vel impar numerus.

Sit $x = AP = a$, biſecetur AP in T in duas æquales partes, & ducatur linea ET δ, & ſi AE, EM, AM, jungantur; erit triangulum AEM = TP $\varepsilon\delta$ T areæ.

Deinde,

Deinde, biſecentur T P, A T in R and V, & ducantur R G, C V γ; & jungantur A C, C E, E G, G M; & erunt duo triangula A C E + E G M = V T $\delta\gamma$ V areæ.

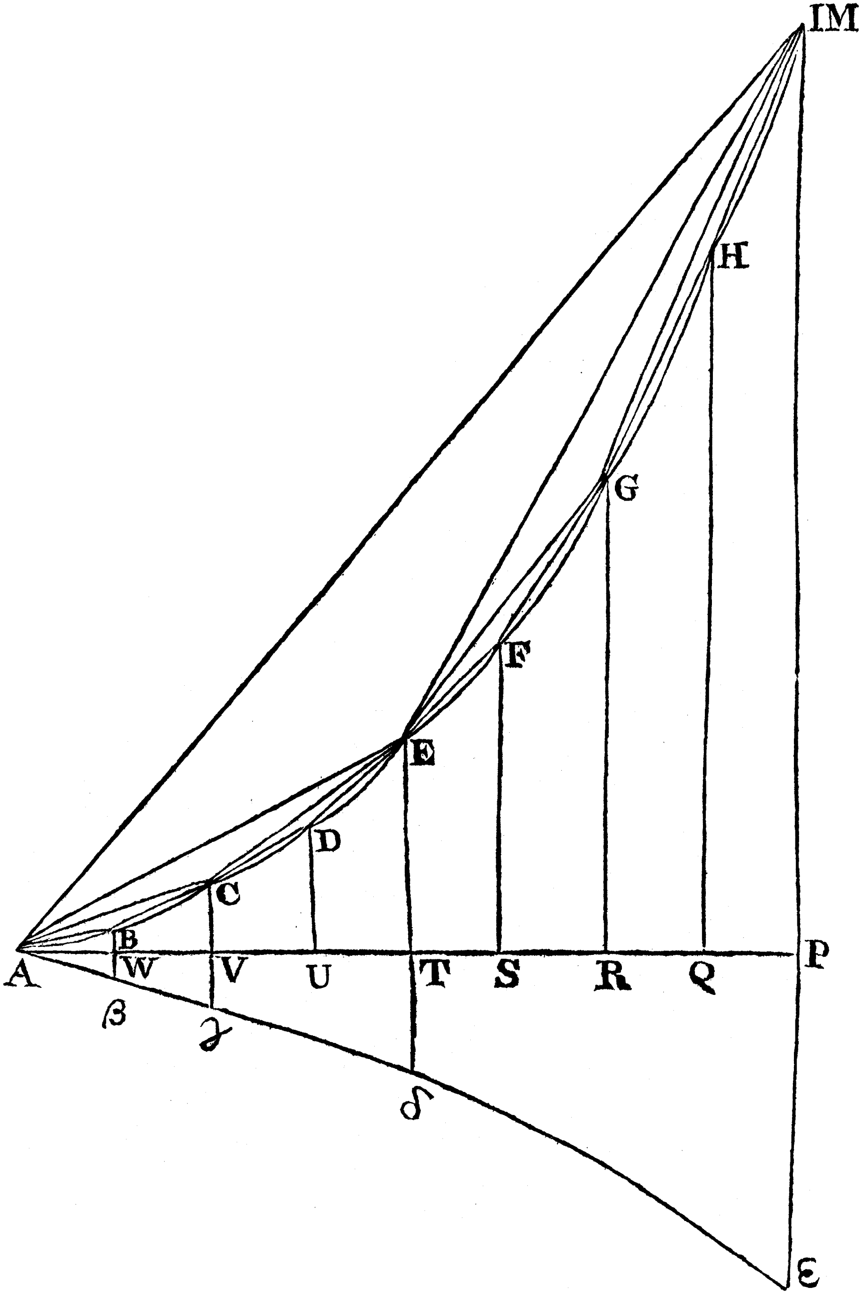
Eodem modo, ſi partes A V, V T, T R, R P iterum biſecentur in W, U, S, Q, & ducantur lineæ B W β, U D, S F, Q H; & jungantur A B, B C, C D, D E, E F, F G, G H, H M; erunt quatuor triangula A B C + C D E + E F G + G H M = W V $\gamma\beta$ W areæ; & ſic deinceps.

Cor. 1. Si curva A B C & M ſit conica parabola, $(c, e)\ y = p i x^2$, erit $v = \frac{1}{3} p a x$; & A $\beta\gamma\delta$ &c. erit recta linea; & propoſitio eadem eſt cum notiſſimâ propoſitione Archimedis de quadraturâ parabolæ.

Cor. 2. Si $y = p x^3$, erit $v = \frac{1}{2} p a^2 x$, & A $\beta\gamma\delta$ &c. iterum recta linea.

Cor. 3. Datâ curvâ, cujus æquatio eſt $y = p x^{2n}$, inveniri poteſt altera curva, cujus dimenſiones ſunt $(2n-1)$, in quâ ſummæ triangulorum ad ſingulas biſectiones erunt reſpectivè æquales ſummis triangulorum datæ curvæ.

His adjici poteſt, quod ſi loco biſectionis abſciſſa A P aliâ quâvis ratione in æquales partes dividatur, ſummæ triangulorum curvæ A B C D &c. ad ſingulas diviſiones æquales erunt ſegmentis curvæ A $\beta\gamma\delta$ &c.



XLVII. *Second*